Topic: Solving with substitution

Question: Find the limit.

$$\lim_{x \to 5} \frac{x^2 + 2x + 1}{x + 5}$$

Answer choices:

A 12

B 6

C 1.6

D 3.6

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Solution: D

The limit as x approaches 5 of

$$f(x) = \frac{x^2 + 2x + 1}{x + 5}$$

can be found by direct substitution, since there are no holes or breaks in the graph of $f(x)$ at $x = 5$.

$$f(5) = \frac{5^2 + 2(5) + 1}{5 + 5}$$

$$f(5) = \frac{36}{10}$$

$$f(5) = 3.6$$

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Topic: Solving with substitution

Question: Find the limit.

$$\lim_{x \to 0} \frac{\frac{1}{x+3} - \frac{1}{3}}{x}$$

Answer choices:

A $-\frac{1}{9}$

B $\frac{1}{9}$

C $-\frac{1}{6}$

D $\frac{1}{6}$

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Solution: A

The limit as x approaches 0 of the function can't be found by direct substitution because we'd get 0 in the denominator when $x = 0$.

However, we have a clue that the zero should divide out because we get $0/0$ by direct substitution.

We'll start by finding a common denominator within the numerator. Since one denominator is $x + 3$ and the other is 3, the common denominator is the least common multiple of those, $3(x + 3)$. To get that common denominator, we'll multiply the first fraction in the numerator by $3/3$ and we'll multiply the second fraction in the numerator by $(x + 3)/(x + 3)$.

$$\lim_{x \to 0} \frac{\frac{1}{x+3} - \frac{1}{3}}{x}$$

$$\lim_{x \to 0} \frac{\frac{1}{x+3}\left(\frac{3}{3}\right) - \frac{1}{3}\left(\frac{x+3}{x+3}\right)}{x}$$

$$\lim_{x \to 0} \frac{\frac{3}{3(x+3)} - \frac{x+3}{3(x+3)}}{x}$$

Now, because we have a common denominator inside of the numerator, we can combine the fractions. Because there's subtraction between the fractions, there will be subtraction between the terms in the numerators, such that the new numerator will be $3 - (x + 3)$.

$$\lim_{x \to 0} \frac{\frac{3 - (x+3)}{3(x+3)}}{x}$$

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Simplify that numerator.

$$\lim_{x\to 0} \frac{\frac{3-x-3}{3(x+3)}}{x}$$

$$\lim_{x\to 0} \frac{\frac{-x}{3(x+3)}}{x}$$

We have the fraction in the numerator, and x in the denominator. Instead of dividing by x, we can multiply be its reciprocal, $1/x$.

$$\lim_{x\to 0} \frac{-x}{3(x+3)} \cdot \frac{1}{x}$$

The x in the numerator of the first fraction will cancel with the x from the denominator of the second fraction, leaving us with just the -1 (combining the negative sign with the 1) in the numerator, and the $3(x+3)$ in the denominator.

$$\lim_{x\to 0} \frac{-1}{3(x+3)}$$

Now just plug in $x = 0$ to evaluate the limit.

$$\frac{-1}{3(0+3)}$$

$$-\frac{1}{9}$$

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Topic: Solving with substitution

Question: Find the limit.

$$\lim_{x \to 6} \left(x^3 + 6 - 3x\right)$$

Answer choices:

A 204

B 198

C 240

D 234

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Solution: A

One of the properties of limits is

$$\lim_{x\to a}\left[f(x)\pm g(x)\right]=\lim_{x\to a}f(x)\pm\lim_{x\to a}g(x)$$

This means that we can break down our limit into smaller pieces. With more complex limits, this property can be extremely useful.

For this problem, we can take our original limit

$$\lim_{x\to 6}\left(x^3+6-3x\right)$$

and break it down to

$$\lim_{x\to 6}\left(x^3+6-3x\right)=\lim_{x\to 6}x^3+\lim_{x\to 6}6-\lim_{x\to 6}3x$$

$$\lim_{x\to 6}\left(x^3+6-3x\right)=\lim_{x\to 6}x^3+\lim_{x\to 6}6-3\lim_{x\to 6}x$$

At this point, we can evaluate our limits

$$\lim_{x\to 6}\left(x^3+6-3x\right)=(6)^3+6-3(6)$$

$$\lim_{x\to 6}\left(x^3+6-3x\right)=216+6-18$$

$$\lim_{x\to 6}\left(x^3+6-3x\right)=204$$

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